Uncertain Productivity Growth and the Choice between FDI and Export

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WEB-APPENDIX

This document provides analytical derivations referred to in the article: *Uncertain Productivity Growth and the Choice between FDI and Export.* Published in the Review of International Economics.

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Appendix

A Equivalent Risk-Adjusted Return

Given the Geometric Brownian motion in equation (22), from Ito's lemma we have:

$$\mathbb{E}\left(\frac{d\phi^{\kappa}}{\phi^{\kappa}}\right) = \left(\kappa\phi^{\kappa-1}d\phi + \frac{1}{2}\kappa(\kappa-1)\phi^{\kappa-2}\sigma^2\phi^2 dt\right)/\phi^{\kappa}.$$

Substituting for $d\phi$ leads to

$$\mathbb{E}\left(\frac{d\phi^{\kappa}}{\phi^{\kappa}}\right) = \left(\alpha\kappa + \frac{1}{2}\kappa\sigma^2(\kappa-1)\right)dt + \kappa\sigma dz_t. \tag{A.1}$$

From the quadratic equation in (B.1), which is valid for ϕ^{κ} with $\Psi(\kappa) = 0$, it follows that

$$\frac{1}{2}\kappa\sigma^2(\kappa-1) = r - (r - (\mu - \alpha))\kappa.$$

Hence, the equivalent risk-adjusted rate of return for an exponential variable results as

$$\mu_e = r + \kappa(\mu - r). \tag{A.2}$$

B The Fundamental Quadratic Equation

Substituting the guess solution $F(i, \phi(\sigma)) = A_1\phi^{\beta}$ and its derivatives into the linear differential equation (27), we receive the fundamental quadratic equation

$$\Psi = \frac{1}{2}\sigma^2\beta(\beta-1) + (r - (\mu-\alpha))\beta - r = 0. \tag{B.1}$$

Consider the total differential

$$\frac{\partial\Psi}{\partial\beta}\frac{\partial\beta}{\partial\sigma} + \frac{\partial\Psi}{\partial\sigma} = 0, \tag{B.2}$$

which can be evaluated at $\beta = \beta_1$. The quadratic equation (B.1) increases in β_1 with $\partial\Psi/\partial\beta_1 > 0$. The derivative of Ψ with respect to σ results as

$$\frac{\partial\Psi}{\partial\sigma} = \sigma\beta_1(\beta_1 - 1) > 0, \tag{B.3}$$

because of (28). From B.2 we have necessarily $\frac{\partial \beta_1}{\partial \sigma} < 0$. Furthermore, the discount rate of periodical profits in equation (25) turns out to be the negative expression of Ψ evaluated at κ. Note that

$$\mu_e - \alpha_e = r - (r - (\mu - \alpha))\kappa - \frac{1}{2}\kappa(\kappa - 1)\sigma^2. \tag{B.4}$$

For bounded results, this discount rate needs to be strictly positive and, hence, κ must lie between the two roots, specifically: $\beta_1 > \kappa > 0$. As a consequence,

$$\frac{\partial \left(\frac{\beta_1}{\beta_1 - \kappa}\right)}{\partial \sigma} > 0. \tag{B.5}$$

For $\sigma = 0$, we have $\mu = r$, and from equation (B.1) it follows that $\beta_1 = \frac{r}{\alpha} = \beta$.

C Expected Entry Time

By using the Girsanov theorem[1] it is possible to derive the probability density function of the waiting time $T(i)$ as

$$f(T(i, \phi^*_\sigma(i)), \phi) = \frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\sqrt{2\pi\sigma^2 T(i, \phi^*_\sigma(i))^3}}\, e^{-\frac{\left(\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right) - (\alpha - \frac{1}{2}\sigma^2)T(i, \phi^*_\sigma(i))\right)^2}{2\sigma^2 T(i, \phi^*_\sigma(i))}} \tag{C.1}$$

with $\phi^*_\sigma(i) > \phi$. The Laplace transform of $T(i, \phi^*_\sigma(i))$ is then given by (see Ross, 1996; Proposition 8.4.1)

$$\mathbb{E}\left(e^{-\lambda T(i, \phi^*_\sigma(i))^*}\right) = \int_0^\infty e^{-\lambda T(i, \phi^*_\sigma(i))} f(T(i, \phi^*_\sigma(i))) dT(i, \phi^*_\sigma(i)) \tag{C.2}$$

$$= e^{-\left(\sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\lambda} - (\alpha - \frac{1}{2}\sigma^2)\right)\frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\sigma^2}} \tag{C.3}$$

and can be used to determine the expected waiting time as

$$\mathbb{E}(T(i, \phi^*_\sigma(i))) = \int_0^\infty T(i, \phi^*_\sigma(i)) f(T(i, \phi^*_\sigma(i))) dT(i, \phi^*_\sigma(i)) \tag{C.4}$$

$$= -\lim_{\lambda \to 0} \frac{\partial \mathbb{E}(e^{-\lambda T(i, \phi^*_\sigma(i))})}{\partial \lambda} = \frac{\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right)}{\alpha - \frac{1}{2}\sigma^2}. \tag{C.5}$$

[1] A detailed derivation is offered by Karatzas and Shreve (1991, p.196) or by Karlin and Taylor (1975, p.363).

More precisely,

$$\mathbb{E}(T_i(\phi^*_\sigma(i),\phi)) = \begin{cases} \frac{1}{\alpha-\frac{1}{2}\sigma^2}\ln\left(\frac{\phi^*_\sigma(i)}{\phi}\right) & \text{if} \quad \alpha > \frac{1}{2}\sigma^2 \\ \\ \infty & \text{if} \quad \alpha \le \frac{1}{2}\sigma^2 \end{cases} \tag{C.6}$$

with $\phi^*_\sigma(i) > \phi$ and $i \in \{E, F\}$.

D Expected Entry Time and Comparative Statics

Exploiting the monotonicity of V_0^e in $\phi^*_\sigma(i)$, we prove that $\frac{\partial \phi^*_\sigma(i)}{\partial \sigma} > 0$ and $\frac{\partial \phi^*_\sigma(i)}{\partial \alpha} < 0$ by proving that $\frac{\partial V_0^e(i,\phi^*_\sigma)}{\partial \sigma} > 0$ and $\frac{\partial V_0^e(i,\phi^*_\sigma)}{\partial \alpha} < 0$, respectively.

Rearranging (36), we obtain

$$\frac{M(i)\phi^*_\sigma(i)^\kappa}{\mu_e - \alpha_e} = V_0^e(\phi^*_\sigma(i)) = \frac{\beta_1}{\beta_1 - \kappa} I(i). \tag{D.1}$$

The derivative of $V_0^e(\phi^*_\sigma(i))$ with respect to σ results as

$$\frac{\partial V_0^e(\phi^*_\sigma(i))}{\partial \sigma} = \frac{\partial \beta_1}{\partial \sigma} I_i \left(\frac{-\kappa}{(\beta_1 - \kappa)^2}\right). \tag{D.2}$$

From B.1 we can derive

$$\frac{\partial \beta_1}{\partial \sigma} = -\frac{\beta_1 \sigma(\beta_1 - 1)}{\sigma^2(\beta_1 - \frac{1}{2}) + r - (\mu - \alpha)}. \tag{D.3}$$

Substituting into D.2 results in

$$\frac{\partial V_0^e(\phi^*_\sigma(i))}{\partial \sigma} = \frac{V_0^e(\phi^*_\sigma(i))\sigma(\beta_1 - 1)\kappa}{(\sigma^2(\beta_1 - \frac{1}{2}) + r - (\mu - \alpha))(\beta_1 - \kappa)}. \tag{D.4}$$

For $\beta_1 > 1$ and $\kappa \ge 1$ the numerator is always positive. We can prove that the denominator is also always positive. To do so, we rewrite (28) as

$$(\beta_1 - \frac{1}{2})\sigma^2 + r - (\mu - \alpha) = \sigma^2 \sqrt{\left(\frac{r - (\mu - \alpha)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 0. \tag{D.5}$$

The right-hand side of this equation is always positive for $\beta_1 > 1$, and hence $\frac{\partial V_0^e(\phi^*_\sigma(i))}{\partial \sigma} > 0$.

Furthermore,

$$\frac{\partial V_0^e(\phi_\sigma^*(i))}{\partial \alpha} = \frac{\partial \beta_1}{\partial \alpha} I_i \left(\frac{-\kappa}{(\beta_1 - \kappa)^2} \right). \tag{D.6}$$

From D.2 we receive

$$\frac{\partial \beta_1}{\partial \alpha} = \frac{-\beta_1}{(\beta_1 - \frac{1}{2})\sigma^2 + r - (\mu - \alpha)} < 0. \tag{D.7}$$

Hence, we have $\frac{\partial V_i^*}{\partial \alpha} > 0$. Since V_i^* behaves as ϕ_i^*, we can state $\frac{\partial \phi_\sigma^*(i)}{\partial \alpha} < 0 \quad \wedge \quad \frac{\partial \phi_\sigma^*(i)}{\partial \sigma} > 0$. With these results we can consider the following partial derivatives of C.6:

$$\frac{\partial \mathbb{E}(T(i, \phi_\sigma^*(i)))}{\partial \sigma} = \frac{\sigma}{\left(\alpha - \frac{1}{2}\sigma^2\right)^2} \ln \left(\frac{\phi_\sigma^*(i)}{\phi} \right) + \frac{1}{(\alpha - \frac{1}{2}\sigma^2)} \frac{1}{\phi_\sigma^*(i)} \frac{\partial \phi_\sigma^*(i)}{\partial \sigma} > 0 \tag{D.8}$$

$$\frac{\partial \mathbb{E}(T(i, \phi_\sigma^*(i)))}{\partial \alpha} = -\frac{1}{(\alpha - \frac{1}{2}\sigma^2)^2} \ln \left(\frac{\phi_\sigma^*(i)}{\phi} \right) + \frac{1}{(\alpha - \frac{1}{2}\sigma^2)} \frac{1}{\phi_\sigma^*(i)} \frac{\partial \phi_\sigma^*(i)}{\partial \alpha} < 0. \tag{D.9}$$

In both modes expected entry time increases in σ and decreases in α.

A longer working paper version of this analysis is published in the discussion papers on Business and Economics No.19/2012 at University of Southern Denmark.